I. Basic course information

- **Title:** Number Theory / Graduate Number Theory
- **Course Description:** Elementary theory of numbers. Divisibility, prime numbers, Fundamental Theorem of Arithmetic, congruences, Diophantine equations, quadratic residues and continued fractions are among topics considered.
- **Prerequisite:** MATH 366 (Instruction to Abstract Algebra), with a grade of C− or better.
- **Instructor:** Prof. Frédéric Latour
- **Credits:** 3.00
- **Office Phone:** 860-832-2855
- **E-mail:** latourfre@ccsu.edu
- **Office:** Marcus White 119
- **Office Hours:**
  - **Wednesdays:** 04:45pm–05:45pm
  - **Thursdays:** 02:15pm–04:00pm & 04:30pm–05:45pm
  - **Fridays:** 01:45pm–02:45pm
  - **Other times by appointment.**
- **Location:** Marcus White 119 and online at [https://us.bbcollab.com/guest/b6723f92afe64dc0a228216a45c69202](https://us.bbcollab.com/guest/b6723f92afe64dc0a228216a45c69202)
- **Textbook:** Elementary Number Theory, by Gareth A. Jones and J. Mary Jones.
- **Class Meeting Times:** Mondays and Wednesdays, 05:55pm–07:10pm, Willard-DiLoreto Hall, room W301.
- **Course Requirements:** Attend and participate in class regularly; complete homework assignments; take quizzes and tests, as scheduled. A general rule for any college course is that you are expected to put in at least 2 hours of work outside of class for every hour in class.
- **Calculator Use:** Calculators will not be needed for this course.
- **Cell phones and other communication devices:** Must be turned off at all times during class. No texting in class.
II. Course Objectives

After taking this course, the student should be able to:

1. Write precise definitions for terms that are used in number theory.
2. State, prove and apply theorems of number theory.

III. Evaluation

Minimum averages have been established for each of these grades:

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<td>87%</td>
<td>77%</td>
<td>67%</td>
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The average for the course will be based on the following weights:

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<tr>
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<th>MATH 469</th>
<th>MATH 569</th>
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<tr>
<td>Homework</td>
<td>20%</td>
<td>16%</td>
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<tr>
<td>Exam 1</td>
<td>25%</td>
<td>20%</td>
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<td>Exam 2</td>
<td>25%</td>
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<tr>
<td>Final Examination</td>
<td>30%</td>
<td>24%</td>
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<tr>
<td>Final Project and Presentation</td>
<td>—</td>
<td>20%</td>
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<tr>
<td>Total</td>
<td>100%</td>
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IV. Schedule of Important Dates

- **Exam # 1**: Wednesday, October 6, in class;
- **Exam # 2**: Wednesday, November 10, in class;
- **Final Examination**: Wednesday, December 8, 05:30pm–07:30pm.

*Please note:* The final examination for this course is cumulative. It covers the entire course.

**Homework** will normally be due on Fridays at 11:59pm. All homework assignments must be submitted electronically (by e-mail); the preferred format is a single PDF file containing your entire homework assignment. You should learn how to use \LaTeX, as it makes typing mathematical solutions much easier, and produces PDF files as output. You are allowed to collaborate with other students on homework, but the solutions that you submit must be your own. Simply copying another student’s work (or allowing another student to copy your work) is considered cheating and is not acceptable. Late homework will be penalized (see VI. Course Policies).

Students in MATH 569 will have to complete a **Final Project**, in which they will be ask to find a theorem of Number Theory that was not covered in class, write a short paper about it, and present in during one of the last three class periods. Details will be provided in the future.

V. University Policies

**Course Accommodations Policy.** Please contact me privately to discuss your specific needs if you believe you need course accommodations based on the impact of a disability, medical condition, or if you have emergency medical information to share. I will need a copy of the accommodation letter from Student Disability Services in order to arrange your class accommodations. Contact Stephanie Scapeccia in Student Disability Services at: 860-832-1952, Willard-DiLoreto Hall, Room W201, if you are not already registered with them. Student Disability Services maintains the confidential documentation of your disability and assists you in coordinating reasonable accommodations with your professors.
Inclement Weather Policy. At the discretion of the President of the University, classes may be cancelled or delayed because of inclement weather conditions or special circumstances. The most accurate cancellation and delay information for Central Connecticut State University will be made available on the Storm Phone: 860-832-3333 and on the Web at www.ccsu.edu.

Course Withdrawal Policy. The last day to withdraw from a course is Wednesday, November 17. Approvals for withdrawal are not required; however, it is strongly recommended that students consult with their instructor and academic advisor prior to deciding to withdraw. Cessation of attendance, notice to the instructor, or telephone calls to the Enrollment Center are not considered official notice of a student’s intention to drop the course.

Beginning on Thursday, November 18, withdrawals are allowed only under extenuating circumstances and require approval of the course instructor and of the Chair of the Department of Mathematical Sciences.

Poor academic performance is not considered an extenuating circumstance.

Basic Needs and CCSU Community Pantry. Any student, faculty or staff who faces challenges securing food, baby items, toiletries, or school supplies and believes this may affect his or her performance in the course can visit our on-campus pantry. Maria’s Place CCSU Community Pantry is available for this purpose. For more information, call the pantry’s office at (860) 832-3662, email the coordinator at mariaspantry@ccsu.edu, or visit our online website at http://compassion.ccsu.edu/foodpantry/.

Statement on Discrimination and Harassment. Central Connecticut State University strives to maintain our campus as a place of work and study for faculty, staff, and students that is free of all forms of prohibited discrimination and harassment based upon age; ancestry, color; gender identity and expression; intellectual disability; learning disability; mental disorder; physical disability; marital status, national origin; race; religious creed; sex, (including pregnancy, transgender status, sexual harassment and sexual assault); sexual orientation; or any other status protected by federal or state laws. Any student who has concerns about this should contact the Office of Equity and Inclusion at 860-832-1653, Student Affairs at 860-832-1601, or his/her faculty member. The Office of Equity and Inclusion is located on the main floor of Davidson Hall, room 119.

Statement on Sexual Misconduct, Intimate Partner Violence, and Stalking. Central Connecticut State University (CCSU) will not tolerate sexual misconduct against students, staff, faculty, or visitors in any form, including but not limited to: sexual assault, sexual exploitation, sexual harassment or stalking, as defined in CCSU policies. For additional information, please consult the CCSU policies at http://www.ccsu.edu/diversity/policies/index.html. All faculty members and staff have a duty to report incidents of sexual harassment, including sexual violence, to the Title IX Officer, Office of Equity and Inclusion, Davidson Hall, room 119.

To file a report, contact: Office of Equity and Inclusion (860-832-1653); Student Affairs (860-832-1601); Student Conduct (860-832-1667) or the University Police (860-832-2375).

For support and advocacy, contact: Office of Victim Advocacy & Violence Prevention (Willard-DiLoreto Hall room D305) at 860-832-3795; Student Wellness Services at 860-832-1945 (confidential), the Women’s Center at 860-832-1655, the local YWCA’s Sexual Assault Crisis Services Hotline at 860-223-1787 (confidential) and Prudence Crandall Center for Domestic Violence (confidential) at 888-774-2900 (24-hour hotline).

Statement on Academic Integrity. All students are expected to demonstrate integrity in the completion of their coursework. Academic integrity means doing one’s own work and giving proper credit to the work and ideas of others. It is the responsibility of each student to become familiar with what constitutes academic dishonesty and plagiarism and to avoid all forms of cheating and plagiarism. Students who engage in plagiarism and other forms of academic misconduct will face academic and possibly disciplinary consequences. Academic sanctions can range from a reduced grade for the assignment to a failing grade for the course. From a disciplinary standpoint, an Academic Misconduct Report may be filed and a Faculty Hearing Board may impose sanctions such as probation, suspension or expulsion.

For further information on academic misconduct and its consequences, please consult the Academic Misconduct Policy (http://www.ccsu.edu/AcademicIntegrity) and the Student Code of Conduct
(http://web.ccsu.edu/StudentConduct). This policy is rigorously enforced by the Department of Mathematical Sciences.

**Face Mask Policy.** For the Fall 2021 semester, students will be required to wear a face mask at all times when in the classroom. Students who are not wearing a face mask, or who are wearing it incorrectly, will not be allowed in the classroom, and will be asked to leave if they are in the classroom. According to CDC guidelines, face masks must cover your nose and mouth and fit snugly against the sides of your face. For more information, please see https://www.cdc.gov/coronavirus/2019-ncov/prevent-getting-sick/about-face-coverings.html.

VI. COURSE POLICIES

**Resources Available.**

(1) If you need help, take advantage of your instructor’s office hours. Do not wait until just before the first test to do so.

(2) The Learning Center is located in Room D316, Willard-DiLoreto Hall. Free tutoring is available. A schedule for hours the Center is open will be posted soon after the beginning of the semester.

(3) Form a study group with other students in your section. Explaining solutions to homework problems to each other is a good way to learn.

(4) A list of private tutors for hire is available in the math department office, Room 128, Marcus White, 860-832-2835.

**Late Homework Policy.** If you submit written homework after the due date, it will be graded with a penalty equal to a percentage of the points earned, depending on how late the homework is submitted:

- Fewer than seven days late: 15% penalty.
- At least seven days but fewer than fourteen days late: 30% penalty.
- At least fourteen days late, but is handed in on or before the last day of the semester: 50% penalty.
- Handed in after the end of the semester’s final exams: no credit.

**Absence Policy.** It is your responsibility to learn the material that you missed if you are absent from class. Classes will be recorded and posted online, but you will not receive credit for attendance and participation if you miss class and watch it later.

**Academic Dishonesty Policy (cheating).** The following are examples of academic dishonesty:

- Copying another student’s, or a tutor’s homework solutions, copying solutions from the internet, or using online services such as Chegg in any way;
- Using, in any way, an instructor’s solutions manual (an instructor’s solutions manual is for use by your instructor only, and not by students or tutors;
- Having an unauthorized formula sheet accessible or visible to you during an examination (whether or not you plan on using it);
- Writing formulas on your hand.

If you are thinking of doing anything that you think is ethically ambiguous, you should ask your instructor if it is cheating. There is no penalty for asking!

The following are NOT cheating:

- Getting help from another student on homework, or working with another student on homework problems, provided that all students write their solutions individually;
- Getting homework help from tutors (Learning Center or private tutors), your professor, or another instructor, provided that you write your solution by yourself.
VII. Course Material

I am planning on covering the material contained in the following sections of the textbook:

Chapter 1: Divisibility
1.1 Divisors
1.2 Bezout’s identity
1.3 Least common multiples
1.4 Linear Diophantine equations

Chapter 2: Prime Numbers
2.1 Prime numbers and prime-power factorizations
2.2 Distribution of primes
2.3 Fermat and Mersenne primes
2.4 Primality-testing and factorizations

Chapter 3: Congruences
3.1 Modular arithmetic
3.2 Linear congruences
3.3 Simultaneous linear congruences
3.4 Simultaneous non-linear congruences
3.5 An extension of the Chinese remainder theorem

Chapter 4: Congruences with a Prime-power Modulus
4.1 The arithmetic of \( \mathbb{Z}_p \)
4.2 Pseudoprimes and Carmichael numbers
4.3 Solving congruences modulo \( p^e \)

Chapter 5: Euler’s Function
5.1 Units
5.2 Euler’s function
5.3 Applications of Euler’s function

Chapter 6: The Group of Units
6.1 The group \( U_n \)
6.2 Primitive roots
6.3 The group \( U_{p^e} \), where \( p \) is an odd prime
6.4 The group \( U_{2^e} \)
6.5 The existence of primitive roots
6.6 Applications of primitive roots
6.7 The algebraic structure of \( U_n \)
6.8 The universal exponent

Chapter 7: Quadratic Residues
7.1 Quadratic congruences
7.2 The group of quadratic residues
7.3 The Legendre symbol
7.4 Quadratic reciprocity
7.5 Quadratic residues for prime-power moduli
7.6 Quadratic residues for arbitrary moduli

Chapter 8: Arithmetic Functions
8.1 Definition and examples
8.2 Perfect numbers
8.3 The Möbius Inversion Formula
8.4 An application of the Möbius inversion formula
8.5 Properties of the Möbius function
8.6 The Dirichlet product

Chapter 9: The Riemann Zeta Function
9.1 Historical background
9.2 Convergence
9.3 Applications to prime numbers
9.4 Random integers
9.5 Evaluating \( \zeta(2) \)
9.6 Evaluating \( \zeta(2k) \)
9.7 Dirichlet series
9.8 Euler products
9.9 Complex variables

Chapter 10: Sums of Squares
10.1 Sums of two squares
10.2 The Gaussian integers
10.3 Sums of three squares
10.4 Sums of four squares
10.5 Digression on quaternions
10.6 Minkowski’s Theorem

* indicates sections that are optional or supplementary.
Chapter 11: Fermat’s Last Theorem

11.1 The problem
11.2 Pythagoras’s Theorem
11.3 Pythagorean triples
11.4 Isosceles triangles and irrationality
11.5 The classification of Pythagorean triples
11.6 Fermat
11.7 The case $n = 4$
11.8 Odd prime exponents
11.9 Lamé and Kummer
11.10 Modern developments
11.11 Further reading

The sections marked with an asterisk are optional and will be covered if time permits.

VIII. Academic Calendar

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