I. Course Title: Problem Solving I, II, III: Math 120, 220, 320 1 credit

II. Course Details:

Section: Tuesdays 4:30 PM – 6:10 PM     MS 223
Instructor: Dr. R. S. Kalder
e-mail: kalderr@ccsu.edu
Phone 860-832-2842
Office: Marcus White 129

Hours: 11:00 am – 4 pm MWR
       12:30 pm – 4 pm T, and by appointment

III. Students for Whom the Course is Intended:

Secondary Mathematics (B.S.) majors. This is one of three one-credit seminars (MATH 120, MATH 220, and MATH 320), which are required for the B.S. Secondary Mathematics majors. It is also recommended for B.S. Elementary Mathematics majors to be taken as an elective.

IV. Attendance:

Since this is a seminar, which meets only once a week and interaction among students is an essential part of the course, attendance at every class is expected. Absences will adversely affect the class participation portion of the final grade.

V. Prerequisites:

For Math 120: Math 115 (C- or higher) or Math 119 (C- or higher) or Placement Exam.
For Math 220: Math 120 (C- or higher) and Math 152 (C- or higher).
For Math 320: Math 220 (C- or higher) and Math 228 (C- or higher).

VI. Textbook:

There is no textbook for this course. Your professor will provide you with handouts containing problems to be worked on. In addition, a list of resources for additional problems is attached to this syllabus.

VII. Work load:

The norm for university courses is that a minimum of two hours of homework per week is required for every credit hour. Consequently, for this class you are expected to spend at least 2 hours per week outside of class.
VIII. Expectations:

All three seminars, MATH 120, MATH 220, and MATH 320 meet at the same time. Students in MATH 220 and MATH 320 will have had the experience of taking this seminar at least once previously. In addition, in most cases they will have completed more of the standard courses in the major. For instance, all students in MATH 220 will have at least one semester in calculus. Students in MATH 320 will have completed courses in discrete mathematics and linear algebra. The mixing of the three groups is deliberately designed to enable students in MATH 120 to learn from more experienced problem solvers.

Because of the heterogeneous nature of the class, expectations are set on an individual basis. You will be given a wide variety of problems to choose from. Work on those that you find challenging, but not impossible. The problems will be weighted according to the level of difficulty. The most straightforward problems will be worth 1 point each, more involved problems 2 points each and so on.

IX. Portfolios:

On four occasions during the semester, you will submit a portfolio containing detailed solutions to some problems you have worked over the past few weeks. The problems submitted must carry a total weight of at least 6 points for Math 120 students, 7 points for Math 220, and 8 points for Math 320. Along with each solution you will reflect on the process you went through to solve the problem, using Polya's four steps as a framework. We are particularly interested in problems that prove challenging to you, those for which your initial approach was unfruitful, and situations that lead to significant insights and discoveries. The problems submitted in the portfolio may include some you have worked on by yourself and some you have discussed with classmates. Please acknowledge instances where other class members or the instructor have contributed to the solution or where you have used hints provided by the author of the textbook or other sources. Portfolios will be assessed for the level of difficulty of the problems, the accuracy and elegance of the solutions, and your analysis of the problem solving process. Note: The nature of the problems given DO NOT lend themselves to calculator use, so likely if you are using a calculator, you are not solving the problem in an efficient way, and your solution may not be acceptable. All portfolio solutions should be typed. You must use the Equation Editor and Geometer’s Sketchpad or Geogebra for a professional looking document. A sample-grading sheet is attached to this syllabus.

Each portfolio must be stapled with a cover sheet that contains the following information:

Your name
MATH 220
Problems submitted:  #4  2 points
                     #10  2 points
                     #27  3 points
        total    7 points

ALSO:
Begin each problem by typing the problem at the top of a new page. Place the problem solutions in numerical order. Also submit your grade sheet with each portfolio (page 7 of this syllabus.)
X. Final Presentation:

In place of a final examination, you will choose one problem to present to the class on April 21, April 28, or May 4, 2020. The problem you choose may come from any of the four problem sets that you will be given this semester. You will give a ten-minute oral presentation explaining your solution. A rubric for the presentation evaluations is provided on the last page of this syllabus.

XI. Class participation:

Your class participation will be graded. An important part of this grade will be oral presentations of problems that carry at least five points and are not included in your portfolios. For these presentations you must choose a question from one of the first three problem sets. You are advised to make these presentations early and throughout the course, and must complete these presentations no later than April 14, 2020.

XII. Evaluation:

Your grade for the course will be determined on the following basis:

<table>
<thead>
<tr>
<th>Component</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class participation</td>
<td>20%</td>
</tr>
<tr>
<td>Portfolios (four, 15% each)</td>
<td>60%</td>
</tr>
<tr>
<td>Final presentation</td>
<td>20%</td>
</tr>
<tr>
<td>Total</td>
<td>100%</td>
</tr>
</tbody>
</table>

Calendar

Tuesday, January 21, 2020 class, introductory thoughts about problem solving
Tuesday, January 28, 2020 class, introduction to Equation Editor and Geometer’s Sketchpad or Geogebra for portfolio preparation
Tuesday, February 4, 2020 class
Tuesday, February 11, 2020 Portfolio #1 Due
Tuesday, February 18, 2020 class
Tuesday, February 25, 2020 class
Tuesday, March 3, 2020 Portfolio #2 Due
Tuesday, March 10, 2020 class
Tuesday, March 24, 2020 class
Tuesday, March 31, 2020 Portfolio #3 Due
Tuesday, April 7, 2020 class
Tuesday, April 14, 2020 class
Tuesday, April 21, 2020 Final Presentations begin, Portfolio #4 Due
Tuesday, April 28, 2020 Final Presentations
Tuesday, May 5, 2020 Final Presentations
University Policies:

1. If you need course adaptations or accommodations because of a disability, if you have emergency medical information to share with me, or if you need special arrangements in case the building must be evacuated, please make an appointment with me as soon as possible. My telephone numbers and office hours are given above.

2. In the event of a weather emergency, which requires curtailment or cancellation of classes, listen to WTIC (1080 AM) or call (860) 832-3333 for the "general snow message." In the event that I am prevented from traveling to campus and must cancel class, I will send the class an e-mail message. Last day to drop the course without permission is April 22. Withdrawal forms are available in the Enrollment Center, Willard Hall. Cessation of attendance, notice to the instructor, or telephone calls to the Enrollment Center are not considered official notice of a student's intention to withdraw from the course.

Resources:

1. Works about Problem Solving and the Teaching or Problem Solving

Polya, George. How to Solve It

Make Sense of Problems and Persevere in Solving Them

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jump into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?”

MP: That brings up my next question. How did you become interested in heuristics and the art of problem solving? Did anyone or any event influence you on this?

Polya: Well, I think I wrote it somewhere. In one of my books it is mentioned. I came very late to mathematics. I had an interest in biology, literature and philosophy. And as I came to mathematics and learned something of it, I thought: Well, it is so, I see, the proof seems to be conclusive, but how can people find such results? My difficulty in understanding mathematics: How was it discovered? And then I was deeply influenced by some books. I wish to mention just two. One was the book of Ernst Mach on the history of mechanics. For me personally this was the most beautiful book I read. I read it at the right time because I knew a little physics, but just a little. I was just right for it. His main theme is: You cannot understand a theory unless you know how it was discovered. His best book and best-known book is on mechanics, but he wrote also other books, on the theory of heat and still others. But that was the main idea: In order to understand a theory really, you must know how it was discovered. So he came to heuristics. In fact, in some of his other books there are a few direct remarks on problem solving. Then I thought about it and I came across the Regulae of Descartes, which is really a book on problem solving. That is not mentioned in any history of philosophy, because those historians who wrote about him didn't know about problem solving. My interest in literature contributed a little. When I was interested in literature, I was most interested in books of Hippolyte Tame and he wrote about literature in a quasi-scientific way. How in such a vague subject you can bring in something that approximates science, that deeply impressed me. It also contributed to my interest in heuristics. It is essentially a vague question, and that you can introduce something which has something to do with science, that I think I learned from Tame. I was also impressed by his style.

George Polya was interviewed by G.L. Alexanderson at Polya's 90th birthday party at Stanford, May 13, 1977. You can read more about this interview in *Mathematicatical People: Profiles and Interviews*. Burkhauser Boston, Inc., 1985. The pictures are from this source too.
Polya’s problem solving steps:

First. You have to understand the problem.

UNDERSTANDING THE PROBLEM
What is the unknown? What are the data? What is the condition? Is it possible to satisfy
the condition? Is the condition sufficient to determine the unknown? Or is it insufficient?
Or redundant? Or contradictory? Draw a figure. Introduce suitable notation. Separate the
various parts of the condition. Can you write them down?

Second. Find the connection between the data and the unknown.

DEVISING A PLAN
Have you seen it before? Or have you seen the same problem in a slightly different form?
Do you know a related problem? Do you know a theorem that could be useful? Look at
the unknown! And try to think of a familiar problem having the same or a similar
unknown. Here is a problem related to yours and solved before. Could you use it? Could
you use its result? Could you use its method? Should you introduce some auxiliary
element in order to make its use possible? Could you restate the problem? Could you
restate it still differently? Go back to definitions.
If you cannot solve the proposed problem try to solve first some related problem. Could
you imagine a more accessible related problem? A more general problem? A more
special problem? An analogous problem? Could you solve a part of the problem? Keep
only a part of the condition, drop the other part; how far is the unknown then determined,
how can it vary? Could you derive something useful from the data? Could you think of
other data appropriate to determine the unknown? Could you change the unknown or the
data, or both if necessary, so that the new unknown and the new data are nearer to each
other? Did you use all the data? Did you use the whole condition? Have you taken into
account all essential notions involved in the problem?

Third. Carry out your plan.

CARRYING OUT THE PLAN
Carrying out your plan of the solution, check each step. Can you see clearly that the step
is correct? Can you prove that it is correct?

Fourth. Examine the solution obtained.

LOOKING BACK.
Can you check the result? Can you check the argument? Can you derive the result
differently? Can you see it at a glance? Can you use the result, or the method, for some
other problem?

Problem solving heuristics from George Polya, Princeton University Press, 1973
Mathematics 120/220/320 Portfolio Assessment

Name__________________________________

<table>
<thead>
<tr>
<th>Assessment</th>
<th>Possible Points</th>
<th>Student’s points on Portfolios #1</th>
<th>#2</th>
<th>#3</th>
<th>#4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. The portfolio is complete in that it contains the solutions of problems with a total number of points appropriate to the level of the course (6 for 120, 7 for 220, 8 for 320).</td>
<td>10</td>
<td>_____</td>
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<tr>
<td>2. Solutions to the problems are accurate and complete.</td>
<td>15</td>
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<td>3. The student uses Polya’s four steps effectively to solve problems.</td>
<td>15</td>
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<td>4. The student demonstrates significant insight and creativity.</td>
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<td>5. The selection includes items that were challenging to the student and varied in topics.</td>
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<td>6. The writing is clear and easy to understand.</td>
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<td>7. The writing is free of errors in the use of mathematical terminology.</td>
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<tr>
<td>8. The writing is free of grammatical and spelling errors.</td>
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<tr>
<td>Total</td>
<td>100</td>
<td>grade</td>
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Problem Solving Evaluation of Presentation

Presenter: Score: _______ of 100

1. 60 points Application of Polya’s problem solving strategies
   a. _______ Understanding the problem (15 points)
      i. _______ What is the unknown?
      ii. _______ What are the conditions?
      iii. _______ Are the conditions sufficient?
      iv. _______ Draw a figure
      v. _______ Introduce notation
   b. _______ Devise a plan (15 points)
      i. _______ Have you seen it before? Or in a slightly different form?
      ii. _______ Do you know a useful theorem?
      iii. _______ Do you know a related problem?
      iv. _______ Did you include all the data? All the conditions?
   c. _______ Carry out the plan (15 points)
      i. _______ Followed the plan
      ii. _______ Check that each step is done correctly?
      iii. _______ Can you prove that your solution is correct?
   d. _______ Looking back (15 points)
      i. _______ Can you check your result?
      ii. _______ Can you check the argument?
      iii. _______ Is there another way to see the result at once? (elegance)
      iv. _______ Can you use the result or the method for some other problem?

2. 40 points Presentation methods
   a. _______ Visual resources (looking for effectiveness of use, not a particular resource.) Some options are listed below:
      i. Chalk board
      ii. Overheads
      iii. Handouts
      iv. Models
      v. Other
   b. _______ Timing planned effectively
   c. _______ Voice pleasant, easy to hear
   d. _______ Used correct Mathematical terminology and English grammar
   e. _______ Aware of the audience
   f. _______ Presentation planned to respond to the interest and ability level of the audience

3. General Comments